

4/MTH-251 Syllabus-2023

2026

(May-June)

FYUP : 4th Semester Examination

MATHEMATICS

(Differential Equations)

(MTH-251)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Answer **four** questions, selecting **one** from each Unit.

UNIT—I

1. (a) Solve :

$$\frac{dy}{dx} = \sin(x - y) \quad 4$$

(b) Solve any *three* of the following : $4 \times 3 = 12$

(i) $(x^2 + y^2)dx - 2xydy = 0$

(ii) $(x + y)(dx - dy) = dx + dy$

(2)

(iii) $y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$

(iv) $(1 - x^2) \frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$

(c) Show that the equation

$$y \sin 2x dx - (y^2 + \cos^2 x) dy = 0$$

is exact.

3

2. (a) Obtain the complete primitive and singular solution of $y = px + \sqrt{1 + p^2}$.

5

(b) Solve :

5

$$p^2 y + (x - y)p - x = 0$$

(c) Solve :

5

$$(1 + x^2) \frac{dy}{dx} = 1 + xy \frac{dy}{dx}$$

for which $y = 2$ when $x = 0$.

(d) Solve :

4

$$(x + y + 1)dx - (2x + 2y + 1)dy = 0$$

UNIT—II

3. (a) Solve :

3

$$\frac{d^3 y}{dx^3} - 4 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} - 2y = 0$$

(3)

(b) Solve any three of the following : $5 \times 3 = 15$

(i) $(D^2 + 6D + 9)y = 2e^{-3x}$

(ii) $(D^2 + D + 1)y = \sin 2x$

(iii) $(D^2 - 4D + 4)y = x^3$

(iv) $(D^2 - 4D + 3)y = e^{2x} \sin 3x$

4. (a) Solve :

3

$$\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - \frac{dy}{dx} + y = 0$$

(b) Solve any three of the following : $5 \times 3 = 15$

(i) $(D^2 - 2D + 5)y = e^x \cos 2x$

(ii) $(D^2 + 2D + 1)y = 2x + x^2$

(iii) $(D^2 + 9)y = \cos 2x + \sin 2x$

(iv) $(D^2 - 5D + 6)y = e^{2x}$

UNIT—III

5. (a) Solve :

5

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$$

- (b) Prove that the equation

$$(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \sec^2 x$$

is exact and hence solve the equation. 4

- (c) Reduce the equation

$$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = xe^x$$

to the normal form and hence solve it. 5

- (d) Solve the equation

$$x^6\frac{d^2y}{dx^2} + 3x^5\frac{dy}{dx} + a^2y = \frac{1}{x^2}$$

by changing the independent variable. 5

6. (a) Solve the following simultaneous differential equation : 5

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}$$

- (b) Apply the method of variation of parameters to solve the equation : 5

$$\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$$

- (c) Solve : 5

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

- (d) Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$$

is integrable. 4

UNIT—IV

7. (a) Form a partial differential equation by eliminating
- h
- and
- k
- from the equation

$$(x-h)^2 + (y-k)^2 + z^2 = \lambda^2 \quad 4$$

- (b) Form a partial differential equation by eliminating the function
- f
- from the equation

$$lx + my + nz = f(x^2 + y^2 + z^2) \quad 5$$

- (c) Solve : 5

$$x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$$

- (d) Find the equation of the integral surface of the differential equation

$$2y(z-3)p + (2x-z)q = y(2x-3)$$

which passes through the circle $z=0$, $x^2 + y^2 = 2x$. 5

8. (a) Find the complete and singular integrals of
- $z = px + qy - 2\sqrt{pq}$
- . 2+4=6

- (b) Find the complete and general solution of the equation $p^2 + q^2 = m^2$, where m is a constant. Also find the singular solution if it exists. 4+4=8
- (c) Find the complete integral by Charpit's method : 5

$$p^2 - y^2q = y^2 - x^2$$

★ ★ ★